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BOOK of ABSTRACTS

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The spectrum of a periodic dimer near the bottom of the potential

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We consider a two-dimensional Schrödinger operator with the potential which is periodic in one variable and raises at infinity in the other. The variables do not separate. Such operators discribe so-called dimers moving on a periodic substrate [1].

We study the semi-classical asymptotics of the part of the spectrum near the global minimum of the potential (the well). We show that it has a band structure similar to a one-dimensional Schrödinger (or Sturm-Liouville) operator. We calculate asymptotic formula for the widths of bands and the asymptotics for dispersion relations, i.e. formulas connecting energy and quasimomentum. Like in the case a two-dimensional operator with a double well potential (see e.g. [2]) asymptotic formulas appeal to trajectories of a classical Hamiltonian system with the inverted potential. Namely, those trajectories are doubly asymptotic ones to unstable equilibria and librations, i.e. periodic trajectories with velocity vanishing twice on a period. We discuss in detail different situations connected with the behaivior of global minima of the potential, corresponding librations and their influence to the spectrum. This work was prepared in collaboration with S. Yu. Dobrokhotov and A. Fasolino. The work was supported by RFBR grant 14-01-00521.

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Periodic solutions of singularly perturbed Hamiltonian system

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We consider nonintegrable Hamiltonian system with two degrees of freedom with singular perturbation at the origin of coordinate subspace. Two limiting problems of the original one are constructed, each of which is integrable. These problems describe the phase flow dynamics far away from the singularity and near the singularity correspondingly. The arc-solutions, which are such solutions of the limiting problems that start and finish at the origin — the singular point of the perturbative function, are obtained. The set of generating solutions (see [1, 2] for details) of families of periodic orbits is described in terms of obtained arc-solutions.

The generating solution makes it possible to predict such properties of corresponding family of periodic orbits as type of symmetry, global multiplicity of the orbit of solution and first approximation of initial conditions of the solution. An algorithm for investigation of families of periodic solutions over its generating sequence was constructed [3]. This algorithm is applied to finding families of symmetric periodic obits of the well-known celestial mechanics problem – Hill problem. More than thirty new families of periodic solutions with different types of symmetry were found out and completely described [4]. For continuation of the family of periodic solution a variant of predictor-corrector method is used, which essentially explores the structure of the monodromy matrix of the periodic solution and provides the monitoring of bifurcations of solutions.

It is possible to provide quite natural generalization of the original problem, which include the singular perturbation with the opposite sign. The set of generating solution for such generalized problem can be build as mentioned above. In the case of Hill problem the Hamiltonian function of the generalized one takes form

$$\widetilde{H} = \frac{1}{2}\left(y_1^2 + y_2^2\right) + x_2 y_1 - x_1 y_2 - x_1^2 + \frac{1}{2}x_2^2 + \frac{\sigma}{r},$$

where $r = \sqrt{x_1^2 + x^2}$ and $\sigma = \pm 1$. For value $\sigma = -1$ one gets the Hamiltonian of the classical case. We call the problem with $\sigma = +1$ as anti-Hill problem.

The structure of periodic solutions of anti-Hill problem is considerably simpler then in Hill problem and could be totally described with its generating solutions.

It was shown by numerical computations that all the known families of periodic solutions of Hill problem are continued to periodic solutions of anti-Hill problem but not all anti-Hill problem's families can be continued into Hill problem ones. More over, the further numerical experiment demonstrated that all families of Hill and anti-Hill problems form the common network connecting to each other by common generating solutions and by sharing common orbits with integer multiplicity of different families as well [5].

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Left-handedness: flows that are suspensions inf many ways

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Among 3-dimensional flows, those that can be written as suspensions are particularly interesting, since their study reduces to the study of the first-return map, a 2-dimensional homeomorphism. Sadly, few flows are suspensions. The notion of left-handedness weakens this notion but keeps most of the interesting properties: a flow is left-handed if after removing any finite collection of periodic orbits it is a suspension. We will see some implications of this definition, and study several examples, in particular we will see that many geodesic flows on unit tangent bundles to surfaces are left-handed.

Global instability in the ERTBP

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The (planar) ERTBP describes the motion q of a massless particle (a *comet*) under the gravitational field of two massive bodies (the *primaries*, say the *Sun* and *Jupiter*) with mass ratio μ revolving around their center of mass on elliptic orbits with eccentricity e. The aim of this talk is to show that there exist trajectories of motion such that its angular momentum $G := q \times \dot{q}$ performs arbitary excursions in the region $1 \ll G \ll 1/e$, assuming a small eccentricity and an even smaller mass ratio: $0 < \mu \lll e$. In particular, for any $1 \ll G_1 < G_2 \ll 1/e$, there exist trajectories satisfying $G(0) < G_1$ and $G(T) > G_2$ for some time T, so that there is global instability ("diffusion" is the term usually used) in the angular momentum of this problem.

The diffusive trajectories considered have a small energy, so in terms of position their semimajor axis only changes slightly and it consists of ellipses closer to parabolas.

Such diffusion takes no place in the (planar and circular) RTBP, when the eccentricity vanishes and hence the two primaries revolve along circular orbits, since the RTBP is governed by an autonomous Hamiltonian with two degrees of freedom. It is known that, for a non-zero mass parameter small enough, the RTBP is not integrable, although for large G its chaotic zones have a size which is exponentially small in G. This exponentially small phenomenon adds a first difficulty in proving the global instability of the angular momentum G in the ERTBP for large values of G.

The framework for proving this results consists on considering the motion close to the parabolic orbits of the Kepler problem that takes place when the mass parameter is zero. In other words, studying the *infinity manifold*, which turns out to be an invariant object topogically equivalent to a normally hyperbolic invariant manifold (NHIM). Close to this NHIM, it is possible to define a *scattering map*, which contains the map structure of the homoclinic trajectories to the NHIM. Unfortunately, the inner dynamics inside the NHIM is trivial, which adds a second difficulty, since it cannot be used to be combined with the scattering map to design adequate pseudo-orbits for diffusion. Because of this, we introduce the use of *two* different scattering maps whose combination produces the desired diffusive pseudo-orbits, which eventually give rise to true trajectories of the system with the help of standard shadowing techniques.

This talk is based in a joint work with Vadim Kaloshin, Abraham de la Rosa and Tere M. Seara

Quasiperiodic dynamics of low-dimensional ensemble of oscillators

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In this paper we consider the phase models describing the dynamics of chains and rings of three and four coupled self-oscillatory elements. Focus is on the influence of the type and configuration of the coupling setup on the structure of parameter space. The variation of the coupling parameter allows to study the transition from the chain configuration to the case of global coupling. In this paper we consider both the cases of positive coupling parameter (dissipative coupling) and negative ('active' coupling). The latter case is of interest for laser physics and can be implemented in the array of coupled lasers [1,2], and also in neurodynamics [3,4]. The structure of the space of frequency detuning parameters of the oscillators is studied, the space is two-dimensional in the case of three oscillators, and three-dimensional when there are four. We discuss the configuration of domains of regular oscillations and two-, three- and four-frequency quasiperiodic regimes. System is investigated by the method of Lyapunov exponents charts [5,6]. The shape of the full synchronization area depends on the type of coupling and on the number of oscillators in the system. In the case of a chain of three phase oscillators the area of full synchronization is a parallelogram [7], which lies at the intersection of two strips of two-frequency modes corresponding to the fundamental resonances in the system. Areas of higher order resonant two-frequency modes are fan-shaped structures that diverge from the vertices of a parallelogram and shipped into the area of three-frequency modes. In the case of a ring of three phase oscillators the shape of a region of full synchronization depends on the type of coupling. If coupling is dissipative, then it has the form of an oval, and if active - the six-pointed star. The borders of the area of full synchronization coincide with the saddlenode bifurcation curves of the stable equilibriums corresponding to in-phase and anti-phase synchronization. We discuss the changes in the configuration of these curves at the transition from the chain to the ring. For a chain of the four phase oscillators the domain of full synchronization takes the form of inclined parallelepiped. The faces, edges and vertices correspond to the bifurcations of different codimension. Cross section of a parallelepiped in the case of small fixed values of the third frequency detuning parameter has the form of a hexagon, which with increasing value of the parameter is initially transformed into a triangle, and then disappears. Similarly, in the case of the chain of four phase oscillators, the band corresponding to the two-frequency regime disappears eventually with the growth of one of the parameters of frequency detuning. It turns into a square and a diamond, each located at the intersections of bands of three-frequency regimes. When switching from a chain of four phase oscillators to the ring, the complete synchronization area undergo transformations in a specific manner. Also discussed are the discrete analogs of the systems studied, which are the mappings on the torus. Under such a transformation the dynamics to a certain extent inherits the properties of the flow prototype. However, there are effects, such as resonance Arnold web, that are the consequences of the equations discretization. This work was supported by a grant from the President for the Russian leading scientific schools of Russia NS-1726.2014.2 and RFBR project 14-02-00085.

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The effect of weak dissipation on the system with Arnold's Diffusion

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We investigate the effect of weak dissipation on the hamiltonian system with more than two degrees of freedom. It is well known that in such systems the resonance stochastic layers cross each other forming some web in the phase space. This makes the unlimited diffusion possible for any small values of non-integrable perturbation unlike the systems with 2 degrees of freedom. This diffusion was revealed by V.I. Arnold [1] and is known as Arnold's diffusion. We consider the system of two coupled twist maps:

$$\begin{cases} \varphi_1' = \varphi_1 + I_1 \\ I_1' = \alpha I_1 + \varepsilon \frac{df}{d\varphi_1'}(\varphi_1 + I_1, \varphi_2 + I_2) \end{cases} \quad \begin{cases} \varphi_2' = \varphi_2 + I_2 \\ I_2' = \alpha I_2 + \varepsilon \frac{df}{d\varphi_1'}(\varphi_1 + I_1, \varphi_2 + I_2) \end{cases} \tag{1}$$

with smooth and periodic function.The dynamics of (1) in conservative case (α=1) and for f(I_1,I_2)=1/(cos(φ_1)+cos(φ_2)+const) have been studied in [2]. We investigated the transformations of the actions plane (I_1,I_2) structure by calculating both the orbits of the map (1) with dissipative perturbations and its Lyapunov exponents.

It was found that at low values of dissipation in the system attractors appear that are fixed points and invariant curves. Although attractors are regular the chaotic transition process is observed and its duration depends on the initial position so that the regions with the slowest transition form the square-like lattice.

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Influence of oscillatory ensembles properties to excitation propagation

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We study synchronization in ensembles of locally diffusive coupled Bonhoeffer - Van der Pol oscillators. Individual elements frequencies influence on excitation propagation in one and two dimensional media is investigated. We show that excitation propagation speed depends on frequency mismatch between synchronization frequency and elements' individual frequencies. We study formation of waves with free ends in excitation propagation through the boundary of nonuniform media. Qualitative and quantitative results describing this effect are numerical modeling data and analytical research.

Local linear operator

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Consider an evolutionary PDE

$$u_t = f(u) + Lu,$$

where $u = u(t, x_1, x_2, \ldots, x_k)$, L – linear differential (in $\bar{x}$) operator, f – nonlinear function, $f(0) = 0$. Suppose that we have local stationary solution $u(t, \bar{x}) = u_0(\bar{x})$ for the PDE ($\lim\limits_{\bar{x}\to\infty} u_0(\bar{x}) = 0$). Then the superposition of shifts of such a solution ($\sum_k u_0(\bar{x} + \bar{h}_k)$ often behave as interacting particles if $\bar{h}_k$ are sufficiently large. Mathematically, it means the existence of a finite dimensional central manifold around the superposition. The first (and important) step in showing the existence of such a manifold is study the spectra of the linearized operator around the superposition.

In the talk I plan to present a rather old and general theorem of L. Lerman and myself about linear operators that is useful for the cases described above.

Invariant Characteristics of Attractor and Diffusion Chaos in Reaction-diffusion Equation in the Dumbbell Domain

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We consider a problem of homogeneous cycle stability loss in a distributed dynamical system of reaction-diffusion type in the case when space domain boundary changes while its measure is preserved. We study the case of dumbbell domain. Consider the following equation of reaction-diffusion type with Neumann boundary condition:

$$\dot{u} = \nu D \Delta u + F(u), \quad \left.\frac{\partial u}{\partial \vec{n}}\right|_{\partial\Omega} = 0, \tag{1}$$

where $u = u(t,x) \in \mathrm{R}^m$, $t > 0$, $x \in \mathrm{R}^k$, $1 \leq k \leq 3$, Δ — Laplas operator, and diagonal matrix D is of the form $D = \operatorname{diag}\{d_1, \ldots, d_m\}$, $d_j > 0$, $j = 1, \ldots, m$, parameter $\nu > 0$ is responsible for proportional decreasing of diffusion coefficients. Let $\vec{n}$ be an outward normal for piecewise smooth boundary $\partial\Omega$ of bounded domain $\Omega \subset \mathrm{R}^k$.

Suppose the boundary problem (1) permits a spatially homogeneous cycle $u = u_0(t)$ ($u_0(t+T) \equiv u_0(T)$). Consider the following problem: suppose ν small enough but above the critical value when the homogeneous cycle looses stability. Choose the domain Ω to be a pair of rectangles interconnected with a bridge. The bridge width is a bifurcation parameter of the problem and is changed in such a way that the measure of the domain is preserved. The conditions on chaotic oscillations emergence were studied and the dependence of invariant characteristics of the attractor on the bridge width was constructed. By decreasing the bridge width the homogeneous cycle looses stability and then the spatially inhomogeneous chaotic attractor emerges. For the obtained attractor we compute Lyapunov exponents and Lyapunov dimension and notice that the dimension grows as the parameter decreases but is bounded. We show that the dimension growth is connected with the growing complexity of stable solutions distribution with respect to the space variable.

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Exponentially small splitting of separatrices to whiskered tori with quadratic frequencies

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In this talk we will study the splitting of invariant manifolds of whiskered (hyperbolic) tori with two frequencies in nearly-integrable Hamiltonian systems such that the hyperbolic part is given by a pendulum. We will consider 2-dimensional tori whose frequency ratios are quadratic irrational numbers. Applying the Poincaré-Melnikov method, we provide an asymptotic estimate for the maximal splitting distance, and show the existence of 4 transverse homoclinic orbits to the whiskered tori with an asymptotic estimate for their transversality. These estimates are exponentially small in the perturbation parameter, and the functions in the exponents satisfy a periodicity property. This is a joint work with Amadeu Delshams and Pere Gutiérrez.

О диффеморфизмах Морса-Смейла на локально-тривиальных расслоениях над окружностью

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Доклад посвящен изложению результатов, полученных совместно с В.З. Гринесом.

Диффеоморфизм $f : M^n \to M^n$ связного замкнутого гладкого многообразия M^n размерности n называется *диффеоморфизмом Морса-Смейла*, если его неблуждающее множество Ω_f конечно и состоит только из гиперболических периодических точек, и для любых различных седловых периодических точек $p, q \in \Omega_f$ инвариантные многообразия W^s_p, W^u_q либо не пересекаются, либо пересекаются трансверсально. Обозначим класс сохраняющих ориентацию диффеоморфизмов Морса-Смейла на ориентируемых многообразиях через $MS(M^n)$.

С.Смейл показал, что градиентный поток функции Морса на произвольном многообразии M^n может быть сколь угодно близко апроксимирован (в C^1-топологии) потоком Морса-Смейла, что доказывает существование диффеоморфизмов Морса-Смейла на любом многообразии (например, являющихся сдвигами на единицу времени вдоль траекторий потоков Морса-Смейла).

Диффеоморфизмы Морса-Смейла обнаруживают замечательную взаимосвязь между динамикой и топологией несущего многообразия. Так, в работе [1] показано, что если диффеоморфизм $f \in MS(M^3)$ не имеет гетероклинических кривых, то многообразие M^3 диффеоморфно либо сфере S^3, либо связной сумме многообразий $S^2 \times S^1$. Для градиентно-подобных диффеоморфизмов (диффеоморфизмов Морса-Смейла без гетероклинических точек) на 3-многообразиях с тривиально вложенными сепаратрисами в работе [2] установлены соотношения между структурой множества неблуждающих орбит и разложением Хегора несущего многообразия.

В докладе будут получены достаточные условия, при выполнении которых несущее трехмерное многообразие диффеоморфизма Морса-Смейла является локально-тривиальным расслоением над окружностью. Этот результат позволит в последующем „понизить размерность“ задачи о топологической классификации таких диффеоморфизмов, сведя ее к топологической классификации диффеоморфизмов Морса-Смейла на поверхностях и окружности.

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Realization of gradient-like diffeomorphisms on surfaces by means of automorphisms of three-color graphs

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M. Peixoto [3] introduced "distinguished graph" for topological classification of Morse-Smale flows on surfaces. Basic definitions and links on this topic, see, for example, [1]. A. A. Oshemkov and V. V. Sharko [2] suggested a new topological invariant — three-color graph — for such flows which is simpler for checking of isomorphism than Peixoto's one.

We generalize the construction from [2] and associate with every gradient-like diffeomorphism given on surface a three-color graph.

More precisely. Let G be a class of orientation preserving Morse-Smale diffeomorphisms on two-dimensional manifold M^2, satisfying to the following conditions:

1. for any distinct periodic points p, q of a diffeomorphism $f \in G$ the set $W_p^s \cap W_q^u$ is empty (i.e. f is gradient-like);

2. restriction of the diffeomorphism f to the unstable manifold $W^u(p)$ of any saddle periodic point p preserves orientation of $W^u(p)$.

Automorphism P_f of three-color graph T_f is called *periodic* of period $m \in \mathbb{N}$, if $P_f^m(a) = a$ and $P_f^\mu(a) \neq a$ for natural $\mu < m$ for each vertex a the graph T_f.

Theorem 1. *Two diffeomorphisms f, f' from class G are topologically conjugated iff exists isomorphism $\eta : T_f \to T_{f'}$ such that $P'_{f'} = \eta P_f \eta^{-1}$.*

We introduce the class of admissible three-color graphs equipped by periodic automorphisms and prove the following theorem.

Theorem 2. Let admissible three-color graph T with automorphism P. Then there exists a diffeomorphism $f \in G$, for which there is an isomorphism $\eta : T \to T_f$, such as $P = \eta P_f \eta^{-1}$.

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Mixed dynamics in the Pikovsky-Topaj problem of four coupled rotators

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For the well-known Pikovsky–Topaj model which describes the dynamics of four symmetrycally coupled rotators the main biffuraction of symmetry braking is investigated. Me show that these biffuractions lead to the appearing of a such kind of chaos as *mixed dynamics*. Moreover we explain the reason for the numerically observed asymmetry of the attractor and repeller which appears with the increasing of the coupling parameter.

This is a joint work with Gonchenko S.V.

Localized nonlinear modes for NLS with periodic potential: coding and stability

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Localized nonlinear modes for defocusing NLS equation with periodic potential

$$i\partial_t\Psi(x,t) + \partial_x^2\Psi(x,t) - V(x)\Psi(x,t) - |\Psi(x,t)|^2\Psi(x,t) = 0, \qquad (1)$$

(known also as Gross-Pitaevskii equation with repulsive interactions) are the solutions of Eq.(1) of the form $\Psi(x,t) = e^{-i\mu t}\psi(x)$ that satisfy the condition of localization

$$\lim_{x\to\pm\infty} \Psi(x,t) = 0.$$

The function $\psi(x)$ obeys the second order ODE

$$\psi''(x) + (\mu - V(x))\psi(x) - \psi^3(x) = 0.$$

Recently, complete description of these solutions for some classes of potential $V(x)$ has been presented (G.L. Alfimov, A.I. Avramenko, Physica D 254 (2013) 29–45). This approach allows to code these solutions by words of finite length consisting of symbols of some finite alphabet. This contribution is devoted to the two following issues: (i) given the code, how one can restore the profile of the mode? (ii) is it possible to make a conclusion about the stability of the mode by its code?

Answering the point (i), we present a numerical algorithm for construction of the mode profile by its code. In order to study the point (ii) we explore the linear stability of the nonlinear modes with the simplest codes. The study was fulfilled for the model case $V(x) = A\cos 2x$. Numerical computations show that the modes with alternating symbols in the code are unstable. At the same time it has been shown that even the simplest localized mode with one-symbol code may also reveal oscillatory instability. The stability results have been verified by several different numerical tools including the Evans function method and direct solving of eigenvalue stability problem in Fourier space.

Transient structures in a two-component globally coupled reaction-diffusion system

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We study the transient spatio-temporal structures induced by a weak space-time localized stimulus in an excitable contractile fiber with the two-component globally-coupled reaction-diffusion model. The employed model is based on the two-variable Aliev–Panfilov model for normalized transmembrane potential and effective conductivity of repolarizing ionic channels in the nonoscillatory (excitable) cardiac tissue with the local diffusion coupling and depolarizing current of the stretch-activated ionic channels being included. The various regimes of the excitation propagation are analyzed and the origin of the induced structures is determined for various contraction types (determined by the mechanical fixation of the fiber) and the global coupling strengths. We have identified two main regimes of excitation spreading along the fiber: (i) the common quasi-steady-state propagation and (ii) the simultaneous ignition of the major fiber part and have obtained the analytical estimate for the boundary between the regimes in the parameter space. The new oscillatory regimes have been found for the FitzHugh–Nagumo-like system: (i) the propagation of the soliton-like wave with the boundary reflections and (ii) the clusterized auto-oscillations. The single space-time localized stimulus has been shown to be able to induce extremely long-lasting transient activity as a result of the after-excitation effect when the just excited fiber parts are re-excited due to the global coupling. The results obtained indicate the significant role of the mechanical fixation properties (particularly, the contraction type) and the nessecity to take the respective effects into consideration in similar studies.

On homoclinic bifurcations and chaos in asymmetric Duffing–Van-der-Pol equation

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Time-periodic perturbations of an asymmetric Duffing–Van-der-Pol equation close to an integrable equation with a homoclinic "figure-eight" of a saddle are considered. The Poincaré map on the global cross-section through the period of the perturbation generated by this equation is studied.

In the first part of the report the problem of the existence of a rough homoclinic curve for the Poincaré map is considered. The presence of such a curve depends on the relative position of the invariant manifolds of a saddle fixed point for the Poincaré map and is a sufficient condition for the appearance of complicated behavior of solutions. The constructed bifurcation diagram of the Poincaré map on the parameters plane defines all possible cases of the relative position of the separatrices. The presence of an asymmetric term in the perturbation of the initial equation gives a great variety of homoclinic structures.

In the second part of the report the appearance of the complicated behavior of solutions before the tangency of the invariant manifolds of the saddle fixed point is discussed. The moment of transition from simple dynamics to chaos is studied. If the Poincaré map has several attractors (stable fixed and periodic points), the structure of the boundaries of their attraction basins is investigated. It is established that fractal dimension of these boundaries becomes more than topological one before the tangency of stable and unstable separatrices of the saddle fixed point. Therefore, the obtained (using the Melnikov criterion) boundaries of birth of homoclinic structures are not boundaries of the transition from simple dynamics to complex one. In this case, the moment of occurrence of nontrivial hyperbolic structures coincides with the transition moment of the fractal dimension of boundaries through the unit.

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Some distributed systems with chaotic pattern dynamics associated with Smale-Williams attractors

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In recent years examples of spatially distributed systems were introduced with Smale-Williams solenoid in Poincaré cross-section [1, 2]. The systems operate in such way that wave patterns arise and decay alternately with spatial phase of their Fourier components undergoes an expanding circle map. The spatial phase is characterized by angular variable fitted along a filaments of Smale-Williams solenoid.

We discuss three new models with Smale-Williams attractor described by partially differential equations. The first model is a string with parametric excitation of standing wave patterns [3]:

$$\rho(x)\, \partial_t^2 u = -\left(\alpha + u^2\right) \partial_t u - \gamma u + G(t)\, \partial_x^2 u, \tag{1}$$

Boundary conditions are periodic. The term $-\left(\alpha + u^2\right) \partial_t u$ provides nonlinear dissipation, the term $-\gamma u$ provides dissipation of excitations with wave number $k = 0$. Function $\rho(x) = 1+\varepsilon \sin m k_0 x$ describes distribution of mass along the string. Function $G(t) = 1 + a \cos^2 \frac{\pi t}{T} \sin 2\omega_0 t + b \sin^2 \frac{\pi t}{T} \sin 2n\omega_0 t$ describes tension of the string. Frequency ω_0 and wave number k_0 are chosen equal. Factor m equals to $n + 1$ or $n - 1$.

The variation of the tension pumps standing waves with $k = k_0$ and $k = nk_0$ alternatively. On each successive period of pumping the spatial phase transforms by map $\phi' = \pm n\phi + \text{const} \pmod{2\pi}$ with sign depends on m, thus giving birth of Smale-Williams attractor.

The second system is a modified Brusselator model with space and time modulation of parameters:

$$\begin{cases} \partial_t u = (A - u)\,(1 + \varepsilon \cos 6x) - Bu + u^2 v + \gamma(t)\, \sigma\, \partial_x^2 u, \\ \partial_t v = Bu - u^2 v - \gamma(t)\, \partial_x^2 v. \end{cases} \tag{2}$$

Boundary conditions are periodic. Function $1 + \varepsilon \cos 6x$ in (2) describes inhomogeneity of the medium. Diffusion coefficients $\gamma(t)$ vary periodically from 1 to $1/4$.

Parameters are chosen in such way that Hopf instability is excluded and only Turing patterns appearance is possible. Due to time modulation the spatial Fourier harmonics of wave patterns arise and decay with two different wave numbers $k = 2$ and 4 alternately. Intrinsic quadratic nonlinearity and spatial inhomogeneity provide the excitation transfer between these harmonics in such way that their spatial phases are transformed by doubly expanding circle map $\phi_{n+1} = -2\phi_n + \text{const} \pmod{2\pi}$.

The third system differs from previous two in that it is autonomous [4]:

$$\begin{cases} \partial_t u + \left(1 + \partial_x^2\right)^2 u = \mu u + u^3 - \frac{1}{5} u v^2 + \varepsilon v \cos 3x, \\ \partial_t v = -v + u^2 v + u^2. \end{cases} \tag{3}$$

Boundary conditions are periodic. The first equation in (3) is a modified Swift–Hohenberg equation. The second equation in (3) is auxiliary. Function $\varepsilon \cos 3x$ describes inhomogeneity of the medium. The dynamics consists of sequential birth and death of the spatial patterns. Spatial phases of Fourier components of these patterns on each next stage of activity are transformed according to the doubly expanding circle map $\phi_{n+1} = -2\phi_n + \text{const} \pmod{2\pi}$.

Equations (1), (2) and (3) have been solved numerically. The results of numerical simulations suggest that these systems demonstrate hyperbolic dynamics. Finite dimensional reduced systems for the most significant modes were also obtained. Histograms for distributions of the angles between the stable and unstable subspaces on the attractors of reduced models have been obtained. Histograms for all three models demonstrate that zero values of the angles are absent. So, the test confirms hyperbolicity of the attractor.

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Mixed dynamics in reversible maps with figure-8 homoclinic connections

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In this talk we will study the dynamics and bifurcations of two-dimensional reversible maps with a symmetric saddle fixed point having an asymmetric pair of non-transversal homoclinic orbits (roughly speaking, a figure-8 symmetric non-transversal homoclinic connections). For a one-parameter family of maps unfolding the initial homoclinic tangency, it will be proved the existence of "Mixed Dynamics that is, appearance of cascades of bifurcations related to the birth of asymptotically stable, unstable and elliptic periodic orbits.

Stabilizers of Morse functions on surfaces under the action of symplectic difeomorphisms

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Let (M, ω) be a closed orientable surface endowed with symplectic structure. The group $Symp(M, \omega)$ of symplectic diffeomorphisms of M acts on the space $C^\infty(M)$ of smooth functions on M by the following rule: the result of the action of a symplectic diffeomorphism $h \in Symp(M, \omega)$ on a smooth function $f \in C^\infty(M)$ is just a composition $f \circ h$. Let

$$S(f) = \{f \circ h = f \mid h \in Symp(M, \omega)\}$$

be the stabilizer of f with respect to this action. The aims of this talk is to describe the homotopy types of connected components of $S(f)$ for the case when f is a Morse function.

Критерий топологической сопряжённости 3-диффеоморфизмов с одной орбитой гетероклинического касания

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Результаты получены совместно с О.В. Починкой.

В настоящей работе рассматривается класс Ψ диффеоморфизмов f, заданных на гладких трёхмерных замкнутых ориентируемых многообразиях M^3 и обладающих следующими свойствами:

1) неблуждающее множество Ω_f диффеоморфизма f состоит из конечного числа гиперболических точек;

2) для различных седловых точек $\sigma_1, \sigma_2 \in \Omega_f$ пересечение $W^s_{\sigma_1} \cap W^u_{\sigma_2}$ не пусто только в случае, когда $\dim\ W^s_{\sigma_1} = \dim\ W^u_{\sigma_2} = 2$, при этом оно является трансверсальным всюду, кроме, возможно, одной орбиты касания;

3) окрестность каждой седловой точки допускает C^2-линеаризацию.

Для формулировки основного результата введём следующие обозначения для диффеоморфизма $f \in \Psi$.

Для $i \in \{0,1,2,3\}$ обозначим через Ω_i подмножество Ω_f, состоящее из точек p таких, что $\dim\ W^u_p = i$. Положим $A_f = W^u_{\Omega_0 \cup \Omega_1}$, $R_f = W^s_{\Omega_2 \cup \Omega_3}$, $V_f = M^3 \setminus (A_f \cup R_f)$ и $\hat{V}_f = V_f / f$. Множества A_f, R_f, V_f и $\hat{V}_f$ являются связными, $\hat{V}_f$ является гладким замкнутым 3-многообразием и естественная проекция $p_{_f} : V_f \to \hat{V}_f$ является накрытием, индуцирующим эпиморфизм $\eta_{_f} : \pi_1(\hat{V}_f) \to \mathbb{Z}$. Положим $\hat{\mathbb{W}}^s_f = \bigcup\limits_{p \in \Omega_1} \hat{W}^s_p$ и $\hat{\mathbb{W}}^u_f = \bigcup\limits_{p \in \Omega_2} \hat{W}^u_p$. Компоненты связности $\hat{W}^s_p \subset \hat{\mathbb{W}}^s_f$ и $\hat{W}^u_p \subset \hat{\mathbb{W}}^u_f$ либо не пересекаются, либо пересекаются трансверсально, либо пересекаются нетрансверсально с нарушением условия трансверсальности пересечения в точности в одной точке.

Обозначим через $\mathcal{A}$ множество точек гетероклинического касания. Для любой точки $a \in \mathcal{A}$ обозначим через σ^s_a и σ^u_a седловые точки такие, что a принадлежит пересечению инвариантных многообразий $W^s_{\sigma^s_a}$ и $W^u_{\sigma^u_a}$. Обозначим через μ_a (λ_a) собственное значение точки σ^s_a (σ^u_a) по модулю большее (меньшее) единицы. Положим $\hat{\mathcal{A}} = p_{_f}(\mathcal{A})$. Для $\hat{a} \in \hat{\mathcal{A}}$ положим $\Theta_{\hat{a}} = \frac{\ln \mu_a}{\ln \lambda_a}$. Заметим, что $\Theta_{\hat{a}}$ не зависит от выбора точки в множестве $p_f^{-1}(\hat{a})$. Положим $\hat{C}_f = \{\Theta_{\hat{a}}, \hat{a} \in \hat{\mathcal{A}}\}$.

Определение Набор $S_f = (\hat{V}_f, \eta_f, \hat{\mathbb{W}}^s_f, \hat{\mathbb{W}}^u_f, \hat{C}_f)$ назовем схемой диффеоморфизма $f \in \Psi$.

Определение Схемы S_f и $S_{f'}$ диффеоморфизмов $f, f' \in \Psi$ назовем эквивалентными, если существует гомеоморфизм $\hat{\varphi} : \hat{V}_f \to \hat{V}_{f'}$ со следующими свойствами:

1) $\eta_f = \eta_{f'}\hat{\varphi}_*$;

2) $\hat{\varphi}(\hat{\mathbb{W}}^s_f) = \hat{\mathbb{W}}^s_{f'}$ и $\hat{\varphi}(\hat{\mathbb{W}}^u_f) = \hat{\mathbb{W}}^u_{f'}$;

3) $\Theta_{\hat{a}} = \Theta_{\hat{\varphi}(\hat{a})}$ для $\Theta_{\hat{a}} \in \hat{C}_f$.

Основным результатом данной работы является следующая теорема.

Теорема Диффеоморфизмы $f, f' \in \Psi$ топологически сопряжены тогда и только тогда, когда схемы S_f и $S_{f'}$ эквивалентны.

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On bifurcations in nearly integrable Hamiltonian systems

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We consider bifurcations for nearly integrable Hamiltonian systems with 3/2 degrees of freedom. The bifurcations are related mainly to resonances. Definitions of non-degenerate, degenerate resonances and a degeneration order j are given. The basic attention is given to degenerate resonances in Hamiltonian systems. On the basis of the analysis of average systems are studied bifurcations in neighborhood of degenerate resonances. We distinguish cases when j is even or odd. The results for $j = 2$ and $j = 3$ are presented. The case $j = 2$ is characterized by existence of "vortex pairs", and $j = 3$ by "Karman vortex streets".

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Synchronization in a system of oscillators coupled via periodically and randomly driven common load

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This work aims to investigate the role of fluctuations and periodic forcing in the dynamics of n nonlinear oscillators dynamically coupled by a common linear system (common load) [1]. Particularly, we consider the case where the common load is driven by periodic signal or/and subjected to stochastic force modelled as a white Gaussian noise with zero mean $\langle \xi(t) \rangle = 0$ and with the correlation function $\langle \xi(t)\xi(t+\tau) \rangle = D\delta(\tau)$. It is shown that when the noise intensity D exceeds a certain threshold value D^*, consequential noise-induced synchronization with in-phase oscillations of both the nonlinear oscillators and common linear system is observed [2]. It should be noted that for autonomous system this regime does not exist: the in-phase synchronized oscillators are always in anti-phase synchronized mode with oscillations of the load. In this report we present comparative analysis of synchronous solutions observed for the considered system, for oscillators subjected to a common noise [3], and for oscillators inertially coupled via common load driven by a periodic signal.

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Dynamics of gradient-like 3-diffeomorphism and the characteristic spaces

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We consider gradient-like diffeomorphisms given on closed orientable 3-manifolds M^3. The dynamics of such a diffeomorphism f can be represented as a moving from a connected attractor A_f to a connected repeller R_f. In this case, for manifold $V_f = M^3 \setminus (A_f \cup R_f)$ the space of orbits (the characteristic space) $\hat{V}_f = V_f/f$ is a smooth connected 3-manifold, which in the simplest case (for example, when a diffeomorphism f is embedded to a flow) is the direct product of $\mathbb{S}_g \times \mathbb{S}^1$, where $\mathbb{S}_g$ orientable surface of genus $g \geq 0$. The object of study is the class G_g of gradient-like diffeomorphisms (which are not, in general, a one time shift along the trajectories of a flow) $f : M^3 \to M^3$ for which the characteristic manifold $\hat{V}_f$ is diffeomorphic to $\mathbb{S}_g \times \mathbb{S}^1$. In this paper we prove that any saddle point of the diffeomorphism $f \in G_g, g > 0$ has the positive type of orientation. Also states that the manifold admitting a diffeomorphism $f \in G_g, g \geq 0$ without heteroclinic curves is the connected sum of g copies of $\mathbb{S}^2 \times \mathbb{S}^1$ for $g > 0$ and 3-sphere $\mathbb{S}^3$ for $g = 0$.

The method of generating of the strongly continuous groups of operators leading to the new type of Feynman-like formulas for the linear evolutionary PDEs

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Introduction. Feynman formula is a representation of a solution to the Cauchy problem for a PDE in a form of the limit of the multiple integral where the multiplicity tends to infinity. Having appeard first in the pioneering works of R.P.Feynman on the physical level of rigor, they were extremely useful for physicists as a source of numerical methods in studying the Schrödinger equation. Later mathematicians developed a consistent theory of such formulas and still continue founding more and more applications of Feynman's idea. The history of research in Feynman formulas up to 2009 can be found in [1].

Main result. Suppose we are given a real constant a and a family of bounded self-adjoint operators $(S_t)_{t\geq 0}$ which is Chernoff-equivalent to a (in most interesting cases unbounded) self-adjoint densely defined operator H. Note that Stone's theorem guarantees the existence of the strongly continuous group $(e^{iatH})_{t\in\mathbb{R}}$ with the generator iaH. Then we introduce a short formula $R_t = \exp\left[ia(S_{|t|} - I)\mathrm{sign}(t)\right]$ for another family of (unitary, thanks to the Stone's theorem) operators $(R_t)_{t\in\mathbb{R}}$. We prove that this new family is Chernoff-equivalent to the strongly continuous group $(e^{iatH})_{t\in\mathbb{R}}$. Applying the Chernoff theorem [2] to $(R_t)_{t\in\mathbb{R}}$ we obtain the representation of the group $(e^{iatH})_{t\in\mathbb{R}}$ in the form of Chernoff approximations of a new type. If operators $(S_t)_{t\geq 0}$ are integral operators, then the formulas obtained include both multiple integration (like Feynman formulas) and summation (not like Feynman formulas). Such formulas give us one of the ways to solve the Cauchy problem for the PDE $u'_t(t,x) = iaHu(t,x)$. We call this PDE the general Schrödinger equation, and the group $(e^{iatH})_{t\in\mathbb{R}}$ the general Schrödinger group for two reasons. First, we allow H to be more complicated than a second-order differential (with respect to the spatial coordinate x) operator. Second, we admit that x can range over more complicated spaces than $\mathbb{R}^3$.

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Method of critical subsystems as a way to calculate the types of critical points in integrable systems with three degrees of freedom

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Consider an integrable Hamiltonian system with three degrees of freedom and its integral mapping defined by three functionally independent first integrals in involution. The critical set of this mapping is a union of the so-called critical subsystems, which are almost Hamiltonian systems with less than three degrees of freedom. Critical subsystems are described in two ways. First, they are defined as the sets of critical points lying on the zero level of some naturally arising general first integral. Second, the phase spaces of such critical subsystems are described by the pair of invariant relations. Using this integral and the invariant relations one can explicitly calculate the eigenvalues of the corresponding symplectic operator, thus obtaining the type of critical points belonging to the subsystem with respect to transversal cross-sections (the outer type). If the rank of some critical point is less than two, then it belongs to two or three critical subsystems and the the corresponding outer types form the complete type of the point.

In this talk, we investigate the integrable Hamiltonian system with three degrees of freedom found by V.V. Sokolov and A.V. Tsiganov [1]. This system is known as the generalized two-field gyrostat. We find the pairs of the invariant relations describing invariant 4-dimensional manifolds bearing the critical subsystems which generalize the famous Appelrot classes of critical motions of the Kowalevski top [2].

For each subsystem we point out a commutative pair of independent integrals, describe the sets of degeneration of the induced symplectic structure.

With the help of the obtained invariant relations, for each subsystem we calculate the outer type of its points considered as critical points of the initial integrable system with three degrees of freedom.

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The effect of weak dissipation on the dynamics of degenerate Hamiltonian system

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We investigate the effect of weak dissipation on the dynamics of degenerate in terms of KAM-theorem Hamiltonian systems which are known to have a special structure of phase space called the stochastic web usually [1]. Since the nonlinearly driven harmonic oscillator is known to be the simplest system of this type [1] we consider it with two types of dissipation: linear (1) and van-der-Pole-like nonlinear (2):

$$\ddot{x} - 2\gamma\dot{x} + \omega_0^2 x = -\frac{\omega_0 K}{T}\sin x \sum_{n=-\infty}^{+\infty}\delta(t-nT), \quad (1)$$

$$\ddot{x} - (\gamma - \mu x^2)\dot{x} + \omega_0^2 x = -\frac{\omega_0 K}{T}\sin x \sum_{n=-\infty}^{+\infty}\delta(t-nT). \quad (2)$$

We study the transition to chaos with the increase of nonlinearity parameter K for different resonance orders $q = 2\pi/\omega_0 T$. We have revealed that rigid transitions are typical for practically all cases except of the third-order resonance in the system (2). In this case the infinite cascade of period-doublings and the "crossroad area" structures on the plane "nonlinear parameter K - nonlinear dissipation μ" exist.

Also the bifurcation scenarios of attractors evolution with the increase of dissipation parameters were revealed.

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Diffusion through non-transversal transition chains

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The main goal of our work is to understand the geometric mechanism that gives rise the instability shown by Colliander et al (2010) in the Nonlinear Schrodinger Equation with cubic defocusing. It can be seen as a diffusion mechanism, but it appears that the geometric skeleton of the system is not the standard for Arnold Diffusion: instead of having a sequence of non-resonant invariant tori connected along transverse heteroclinic orbits we have a non-transversal situation.

We expose that the instability (diffusion) can be achieved due to the large dimension of the system, while we try to generate a scheme for this new kind of diffusion that could be applied to other infinitely dimensional Hamiltonian systems.

This is a joint work with Amadeu Delshams and Piotr Zgliczcynski.

Controlled Motions of a Spherical Robot with Pendulum Drive

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Problems of controlling a spherical robot with pendulum drive are considered. A mathematical model of the dynamics of this robot is constructed and control laws in the form of state feedback that provide robot motion along a given trajectory on a horizontal or a non-horizontal plane are synthesized.

A spherical robot is a spherical shell with material bodies inside it. This work is devoted to investigation and synthesis of a control for a spherical robot, driven by a pendulum mechanism. The mathematical model of robot motion has the form

$$\begin{aligned} &(M+m)\ddot{\xi} + ml\ddot{e} = -N\gamma_1 + (M+m)g\gamma_0 + P, \\ &J\dot{\omega} = -Q + r[\gamma_1, P], \\ &ml^2[e,\ddot{e}] + j_0\dot{\Omega} + ml[e,\ddot{\xi}] = Q + mlg[e,\gamma_0], \end{aligned} \tag{1}$$

ξ – position of the center of mass of the spherical shell, ω, Ω – vectors of angular velocity of spherical shell and pendulum, e – unit vector, directed from the center of joint to the center of mass of the pendulum; Q – control torque, P – friction force, N – magnitude of the reaction force orthogonal to the support surface applied to the center of the spherical shell, M и m – masses of sphere and pendulum, J и j_0 – moments of inertia of sphere and pendulum, r – radius of spherical shell, l – distance between center of sphere and center of mass of pendulum. Numerical results of computer simulation that demonstrate efficiency of the proposed control laws are presented.

References:

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Falling motion of a circular cylinder interacting dynamically with a vortex pair in a perfect fluid

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We consider a system which consists of a circular cylinder subject to gravity interacting with N vortices in a perfect fluid. Generically, the circulation about the cylinder is different from zero.

The governing equations are Hamiltonian and admit evident integrals of motion: the horizontal and vertical components of the momentum; the latter is obviously non-autonomous.

We then focus on the study of a configuration of the Föppl type: a falling cylinder is accompanied with a vortex pair ($N = 2$). Now the circulation about the cylinder is assumed to be zero and the governing equations are considered on a certain invariant manifold. It is shown that, unlike the Föppl configuration, the vortices cannot be in a relative equilibrium.

A restricted problem is considered: the cylinder is assumed to be sufficiently massive and thus its falling motion is not affected by the vortices. Both restricted and general problems are studied numerically and remarkable qualitative similarity between the problems is outlined: in most cases, the vortex pair and the cylinder are shown to exhibit scattering.